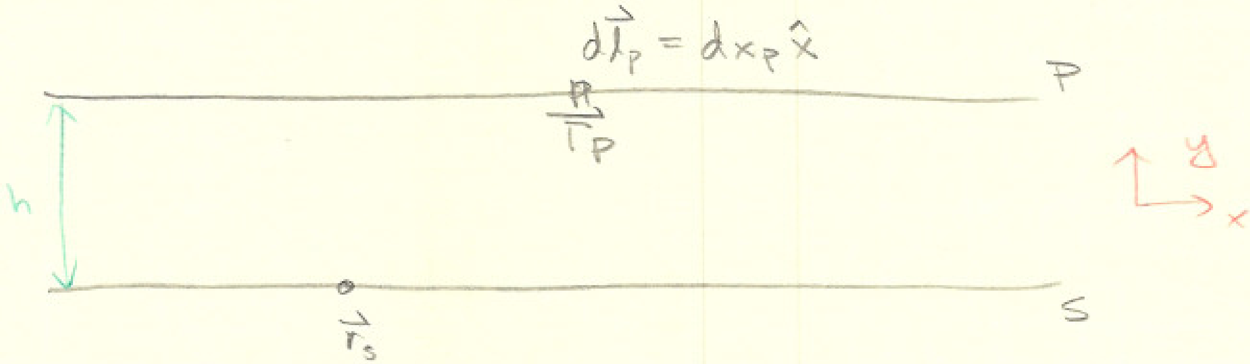
Forces on current carrying wires.

$$\vec{F}_P = \int_{l_s}\int_{l_p} \frac{I_p I_s}{4\pi}\mu_0 \; d\vec{l}_P \times \left(\frac{d\vec{l}_s \times (\vec{r}_p - \vec{r}_s)}{|\vec{r}_p - \vec{r}_s|^3} \right)$$



$$d\vec{l}_s = dx_s\,\hat{x}$$

$$\vec{r}_p - \vec{r}_s = h\hat{y} + \hat{x}\,(\text{something})$$

$$\cancel{d\vec{l}_s = dx_s\,\hat{x}}$$

$$d\vec{l}_s \times (\vec{r}_p - \vec{r}_s) = dx_s\hat{x} \times (h\hat{y} + \hat{x}(\text{something}))$$

$$= dx_s h(\hat{x}\times\hat{y}) = dx_s h\,\hat{z}$$

$$d\vec{l}_p \times (dx_s h\hat{z}) = dx_p dx_s h(\hat{x}\times\hat{z})$$

$$= -\hat{y}\,dx_p\,dx_s h$$

$\vec{F}_p$ is along $-y$

doing the integrals:

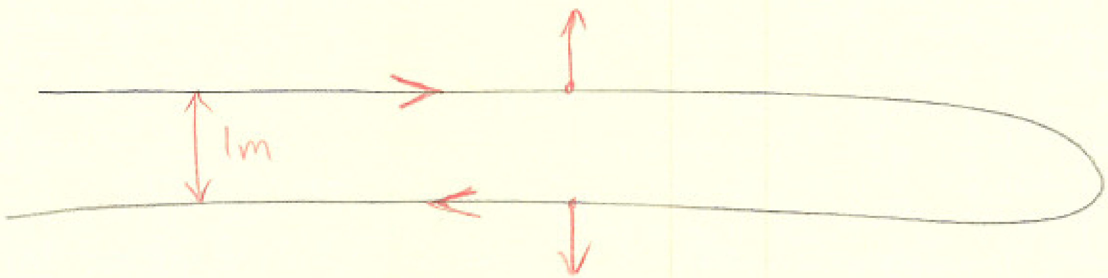
$$\vec{F}_P = l_p \frac{I_p I_s}{2\pi h}\mu_0\,(-\hat{y})$$

$$\vec{F}_s = l_s \frac{I_p I_s}{2\pi h} \mu_0 \left(+\hat{y}\right)$$

$$\frac{\vec{F}_p}{l_p} = \frac{I_p I_s}{2\pi h} \mu_0 \left(-\hat{y}\right)$$

MKSA

meters kg sec Amp

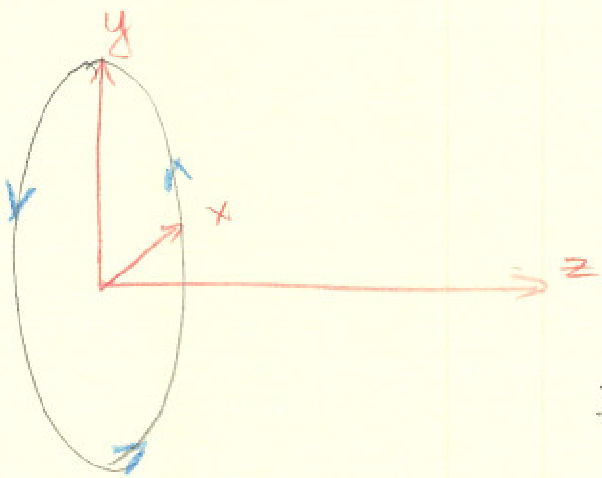


If the currents are in different directions

However if the current is in the same direction they attract.

$\cancel{\vec{F}_p} =$

Current loop and magnetic dipoles



Current loop in the $z=0$ plane.

Find $\vec{B} = \mu_0 \vec{H}$ for a point $\vec{r}_p = z_p \hat{z}$

$$\vec{r}_s = \hat{x}\, r\cos\theta_s + \hat{y}\, r\sin\theta_s$$

$$\vec{H} = \frac{1}{4\pi}\int I\, d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

$$d\vec{l}_s = \hat{x}(-r\sin\theta_s)\,d\theta_s + \hat{y}\, r\cos\theta_s\, d\theta_s$$

$$d\vec{l}_s \times (\vec{r}_p - \vec{r}_s) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -r\sin\theta_s\, d\theta_s & r\cos\theta_s\, d\theta_s & 0 \\ -r\cos\theta_s & -r\sin\theta_s & z_p \end{vmatrix}$$

$$= \hat{x}(z_p r\cos\theta_s\, d\theta_s) - \hat{y}(-r z_p \sin\theta_s\, d\theta_s)$$
$$+ \hat{z}(+r^2\sin^2\theta_s\, d\theta_s + r^2\cos^2\theta_s\, d\theta_s)$$

$$\left.\begin{aligned} H_x &= 0 \\ H_y &= 0 \end{aligned}\right\}\ \text{b/c symm.}$$

$$H_z = \frac{1}{4\pi} I r_p^2 \int_0^{2\pi} d\theta_s = \frac{I\pi r^2}{2\pi}$$

$$\vec{B} = \mu_0 \vec{H} = \hat{z}\,\frac{\mu_0}{2\pi}\underbrace{I\pi r^2}_{\text{Area of the loop} \times I}$$

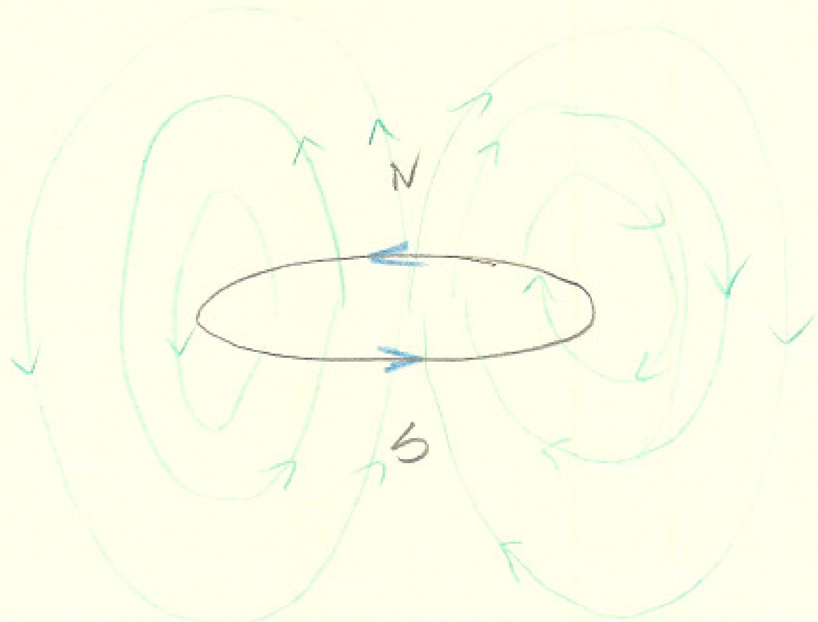
$$|\vec{M}| = (\text{area of loop})\, I$$

this is the magnetic dipole moment

direction of $\vec{H}$ $\perp$ to the area of the loop ; here:

$$\vec{M} = (\pi r^2 I)\hat{z}$$

$$\vec{B} = \frac{\mu_0}{2\pi}\vec{M}$$



This magnetic dipole acts like a magnet

Force on a current loop plunged into a constant $\vec{B}$ feild.

$$\vec{F} = \oint I\, d\vec{\ell} \times \vec{B} = I \cancelto{0}{\oint d\vec{\ell}} \times \vec{B}$$

No net force of a loop in a <u>constant</u> $\vec{B}$ feild.

The fact that the $\overrightarrow{F_{net}} = 0$; There is no effect on $\vec{B}$